AD-A058 804 VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG --ETC F/G 20/4
SECONDARY INSTABILITY IN BOUNDARY-LAYER FLOWS,(U)
AUG 78 A H NAYFEH, A N BOZATLI N00014-75-C-9381

UNCLASSIFIED VPI-E-78-19 NL

1 OF 1
AD
A058804

END
DATE
FILMED
11 78
DDC

12

14

VPI-E-78-19

6

Secondary Instability in Boundary-Layer Flows

10

Ali H. Nayfeh ~~and~~ Ali N. Bozatli

Department of Engineering Science and Mechanics
Virginia Polytechnic Inst~~itute~~ and State Univer~~sity~~
Blacksburg, ~~Virginia~~ 24061

11 Aug[redacted]78

12 35 p.

ONR Contract: NR 061-201
N00014-75-C-9381

15

404 722

mt

Secondary Instability in Boundary-Layer Flows

Ali H. Nayfeh and Ali N. Bozatlı

Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

Abstract

The stability of a secondary Tollmien-Schlichting wave, whose wavenumber and frequency are nearly one half of those of a fundamental Tollmien-Schlichting instability wave (T-S wave), is analyzed by using the method of multiple scales. Under these conditions, the fundamental wave acts as a parametric excitor for the secondary wave. When the amplitude of the fundamental wave is small, the amplitude of the secondary wave deviates slowly from its unexcited state. However, as the amplitude of the fundamental wave increases, so does the amplitude of the secondary wave even in the regions where it is damped in the unexcited state.

I. Introduction

Secondary instabilities in two-dimensional boundary layers over flat plates is analyzed. One of the many disturbances in the flow, a Tollmien-Schlichting instability wave (T-S wave), develops and starts to amplify at a given location on the plate. When this wave grows appreciably downstream, the flow will deviate from the Blasius flow and hence it may be unstable. When this occurs, experimental data shows that the higher harmonics and subharmonics of the principal T-S wave start to amplify faster than the T-S wave itself. It is therefore predicted that the higher harmonics and the subharmonics of the T-S wave play a big role in transition to turbulent flows. This behavior of the disturbances is demonstrated by Kachanov, Kozlov and Levchenko[1] and Veasov, Ginevsky and Karavosov[2] in their experimental works where they excited a known distrubance on a flat plate by a vibrating ribbon and measured the amplitudes of the distrubance spectrum at different downstream locations.

Kachanov, Kozlov and Levchenko[1], in their study of the nonlinear development of a wave in boundary layers, divided the process of the development of a disturbance into four regimes: (1) a linear and slow nonlinear development of distrubances and generation of higher harmonics, (2) an increase in the amplitude of the principal wave, a decrease in the amplitudes of its harmonics and an appearance of low frequencies and subharmonics in the specturm, (3) an interaction of low-frequency oscillations with the principal wave and its higher harmonics, three-dimensional and rapid amplification of all spectral modes and (4) a damping of singled out harmonic modes, smoothing of the spectrum and a transition to the turbulent regime.

Klebanoff, Tidstrom and Sargent[3], in their experimental study of the three dimensional nature of boundary-layer instability, observed the secondary instabilities induced by nonlinear interaction between a two-dimensional fundamental and a three-dimensional secondary wave at a position not very far away from the linear region.

In the present study, we are concerned with the linear and two-dimensional interaction between the fundamental T-S wave and its $\frac{1}{2}$ subharmonic. The wavenumber and the frequency of the subharmonic wave are nearly one half of those of the fundamental wave. The initial amplitude of the fundamental wave is varied and the corresponding amplitudes of the subharmonic wave are calculated at various downstream locations. It is found that the amplitude of the subharmonic is oscillatory in nature and is not amplified appreciably when the fundamental wave is considerably small. As the amplitude of the T-S wave grows downstream, its subharmonic grows in agreement with the experimental observations of Refs. 1 and 2. von Kerczek[4] analyzed the stability of unsteady boundary layers which do not depend on the streamwise coordinate.

II. Problem Formulation

We consider secondary instabilities of a two-dimensional, steady, incompressible flow past a flat plate. The equations describing the motion are the unsteady, dimensionless Navier-Stokes equations:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{R}\nabla^2 U, \tag{2}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{R}\nabla^2 V, \tag{3}$$

$$U = V = 0 \quad \text{at } y = 0, \tag{4}$$

$$U \to U_\infty,\ V \to 0 \ \text{ and } P \to \rho\, U_\infty^2 \ \text{ as } y \to \infty, \tag{5}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Here, x and y are made dimensionless by using a reference length δ_r, the time is made dimensionless by using δ_r/U_∞ and the velocities are made dimensionless by using the freestream velocity U_∞. The Reynolds number is defined as $R = U_\infty \delta_r/\nu$, where ν is the fluid kinematic viscosity.

A. Basic-State

We assume that each basic flow quantity is the sum of a mean flow quantity (the Blasius value) and an unsteady disturbance quantity:

$$\hat{U}(x,y) = U_0(y) + \varepsilon\, U_1(y)\exp[i(kx - \omega t)] + cc \ + O(\varepsilon^2), \tag{6}$$

$$\hat{V}(x,y) = \varepsilon\, V_1(y)\exp[i(kx - \omega t)] + cc \ + O(\varepsilon^2), \tag{7}$$

$$\hat{P}(x,y) = P_0 + \varepsilon\, P_1(y)\exp[i(kx - \omega t)] + cc \ + O(\varepsilon^2), \tag{8}$$

where U_0 and P_0 are mean flow quantities, ε is the amplitude of the T-S wave and cc stands for the complex conjugate of the preceding terms. The dimensionless frequency is represented by ω and the real part of k is the wavenumber whereas the imaginary part of k is the negative of the growth rate. von Kerczek[4] treated the case for which Eqs. (6) - (8) are independent of x.

Substituting Eqs. (6) - (8) into Eqs. (1) - (3), subtracting the mean flow quantities and linearizing the resulting equations in the unsteady disturbance quantities, we obtain the following equations describing the T-S wave:

$$\mathcal{L}_1(U_1,V_1;k) \equiv DV_1 + ikU_1 = 0, \tag{9}$$

$$\mathcal{L}_2(U_1,V_1,P_1;k,\omega) \equiv i(U_0k - \omega)U_1 + V_1DU_0 + ikP_1 - \frac{1}{R}(D^2 - k^2)U_1 = 0, \tag{10}$$

$$\mathcal{L}_3(U_1,V_1,P_1;k,\omega) \equiv i(U_0k - \omega)V_1 + DP_1 - \frac{1}{R}(D^2 - k^2)V_1 = 0, \tag{11}$$

$$U_1 = V_1 = 0 \quad \text{at } y = 0, \tag{12}$$

$$U_1,V_1,P_1 \to 0 \quad \text{as } y \to \infty, \tag{13}$$

where $D \equiv \partial/\partial y$.

B. Stability Analysis

To study the stability of the basic state, we superpose small unsteady disturbances on the basic flow quantities according to

$$U(x,y,t) = \hat{U}(x,y) + \tilde{u}(x,y,t), \tag{14}$$

$$V(x,y,t) = \hat{V}(x,y) + \tilde{v}(x,y,t), \tag{15}$$

$$P(x,y,t) = \hat{P}(x,y) + \tilde{p}(x,y,t), \tag{16}$$

where $\hat{U}$, $\hat{V}$ and $\hat{P}$ represent the basic-state given by Eqs. (6) - (8) and $\tilde{u}$, $\tilde{v}$ and $\tilde{p}$ are the time dependent disturbances which are assumed to be small compared with the basic-state quantities.

Substituting Eqs. (14) - (16) into Eqs. (1) - (5), subtracting the basic-state quantities and linearizing the resulting equations in the unsteady disturbance quantities, we obtain

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0, \tag{17}$$

$$\frac{\partial \tilde{u}}{\partial t} + \hat{U}\frac{\partial \tilde{u}}{\partial x} + \tilde{u}\frac{\partial \hat{U}}{\partial x} + \hat{V}\frac{\partial \tilde{u}}{\partial y} + \tilde{v}\frac{\partial \hat{U}}{\partial y} + \frac{\partial \tilde{p}}{\partial x} - \frac{1}{R}\nabla^2 \tilde{u} = 0, \tag{18}$$

$$\frac{\partial \tilde{v}}{\partial t} + \hat{U}\frac{\partial \tilde{v}}{\partial x} + \tilde{u}\frac{\partial \hat{V}}{\partial x} + \hat{V}\frac{\partial \tilde{v}}{\partial y} + \tilde{v}\frac{\partial \hat{V}}{\partial y} + \frac{\partial \tilde{p}}{\partial y} - \frac{1}{R}\nabla^2 \tilde{v} = 0, \tag{19}$$

$$\tilde{u} = \tilde{v} = 0 \quad \text{at } y = 0, \tag{20}$$

$$\tilde{u},\tilde{v},\tilde{p} \to 0 \quad \text{as } y \to \infty. \tag{21}$$

To determine an approximate solution to Eqs. (17) - (21), we follow the method of multiple scales[5] and seek a uniform expansion in the form

$$\tilde{u} = u_0(x_0,x_1,y,t) + \epsilon u_1(x_0,x_1,y,t) + O(\epsilon^2), \tag{22}$$

$$\tilde{v} = v_0(x_0,x_1,y,t) + \epsilon v_1(x_0,x_1,y,t) + O(\epsilon^2), \tag{23}$$

$$\tilde{p} = p_0(x_0,x_1,y,t) + \epsilon p_1(x_0,x_1,y,t) + O(\epsilon^2), \tag{24}$$

where $x_1 = \epsilon x_0$ is a slow scale. Substituting Eqs. (22) - (24) into Eqs. (17) - (21), using Eqs. (6) - (8) and equating the coefficients of like powers of ϵ, we obtain

Order ϵ^0

$$M_1(u_0,v_0) \equiv \frac{\partial u_0}{\partial v_0} + Dv_0 = 0, \tag{25}$$

$$M_2(u_0,v_0,p_0) \equiv \frac{\partial u_0}{\partial t} + U_0\frac{\partial u_0}{\partial x_0} + v_0 DU_0 + \frac{\partial p_0}{\partial x_0} - \frac{1}{R}(D^2 - \frac{\partial^2}{\partial x_0^2})v_0 = 0, \tag{26}$$

$$M_3(u_0,v_0,p_0) \equiv \frac{\partial v_0}{\partial t} + U_0\frac{\partial v_0}{\partial x_0} + Dp_0 - \frac{1}{R}(D^2 - \frac{\partial^2}{\partial x_0^2})v_0 = 0, \tag{27}$$

$$u_0 = v_0 = 0 \quad \text{at } y = 0, \tag{28}$$

$$u_0,v_0,p_0 \to 0 \quad \text{as } y \to \infty, \tag{29}$$

Order ϵ:

$$M_1(u_1,v_1) = -\frac{\partial U_0}{\partial x_1}, \tag{30}$$

$$M_2(u_1,v_1,p_1) = - U_0\frac{\partial u_0}{\partial x_1} - [U_1\exp(ikx_0 - i\omega t) + cc]\frac{\partial u_0}{\partial x_0} - iku_0[U_1 \times \exp(ikx_0 - i\omega t) - cc] - [V_1\exp(ikx_0 - i\omega t) + cc]Du_0 - v_0[DU_1\exp(ikx_0 - i\omega t) + cc] - \frac{\partial p_0}{\partial x_1} + \frac{2}{R}\frac{\partial^2 u_0}{\partial x_0 \partial x_1}, \tag{31}$$

$$M_3(u_1,v_1,p_1) = - U_0\frac{\partial v_0}{\partial x_1} - [U_1\exp(ikx_0 - i\omega t) + cc]\frac{\partial v_0}{\partial x_0} - iku_0[V_1 \times \exp(ikx_0 - i\omega t) - cc] - [V_1\exp(ikx_0 - i\omega t) + cc]Dv_0 - v_0[DV_1\exp(ikx_0 - i\omega t) + cc] + \frac{2}{R}\frac{\partial^2 v_0}{\partial x_0 \partial x_1}, \tag{32}$$

$$u_1 = v_1 = 0 \quad \text{at } y = 0, \tag{33}$$

$$u_1,v_1,p_1 \to 0 \quad \text{as } y \to \infty. \tag{34}$$

III. Solution

A. Basic-State Solution

The solutions to Eqs. (9) - (13) can be expressed as

$$U_1 = A\zeta_{11}(y), \tag{35}$$

$$V_1 = A\zeta_{12}(y), \tag{36}$$

$$P_1 = A\zeta_{13}(y), \tag{37}$$

where A is a constant and ζ_{1n} (n = 1,2,3) are the eigenfunctions of the parallel problem given by the following equations:

$$\mathcal{L}_1(\zeta_{11},\zeta_{12};k) = 0, \tag{38}$$

$$\mathcal{L}_2(\zeta_{11},\zeta_{12},\zeta_{13};k,\omega) = 0, \tag{39}$$

$$\mathcal{L}_3(\zeta_{11},\zeta_{12},\zeta_{13};k,\omega) = 0, \tag{40}$$

$$\zeta_{11} = \zeta_{12} = 0 \quad \text{at } y = 0, \tag{41}$$

$$\zeta_{11},\zeta_{12},\zeta_{13} \to 0 \text{ as } y \to \infty. \tag{42}$$

B. Zeroth-Order Solution

The solution of the zeroth-order problem given by Eqs. (25) - (29) is taken in the form

$$u_0 = B(x_1)\zeta_{21}(y)\exp[i(Kx_0 - \Omega t)] + cc, \tag{43}$$

$$v_0 = B(x_1)\zeta_{22}(y)\exp[i(Kx_0 - \Omega t)] + cc, \tag{44}$$

$$p_0 = B(x_1)\zeta_{23}(y)\exp[i(Kx_0 - \Omega t)] + cc, \tag{45}$$

where K is the dimensionless propagation constant and Ω is the dimensionless frequency for the secondary mode. The amplitude B is still an undetermined function; it is determined by imposing the solvability condition at the next level of approximation. The quasi-parallel Orr-Sommerfeld problem is

$$\mathcal{L}_1(\zeta_{21},\zeta_{22};K) = 0, \tag{46}$$

$$\mathcal{L}_1(\zeta_{21},\zeta_{22},\zeta_{23};K,\Omega) = 0, \tag{47}$$

$$\mathcal{L}_3(\zeta_{21},\zeta_{22},\zeta_{23};K,\Omega) = 0, \tag{48}$$

$$\zeta_{21} = \zeta_{22} = 0 \quad \text{at } y = 0, \tag{49}$$

$$\zeta_{21},\zeta_{22},\zeta_{23} \to 0 \quad \text{as } y \to \infty. \tag{50}$$

C. First-Order Problem

Since the homogeneous parts of Eqs. (30)-(34) are the same as Eqs. (25)-(29) and since the latter equations have a nontrivial solution, the inhomogeneous Eqs. (30)-(34) have a solution if, and only if, a solvability condition is satisfied. The inhomogeneous parts in Eqs. (30)-(34) contain terms proportional to $\exp[i(kx_0 - \omega t)]$, $\exp[i(kx_0 - \omega t) - i(\bar{K}x_0 - \omega t]$, their complex conjugates and others. Secular terms will arise at this order when $k \simeq 2K$ and $\omega \simeq 2\Omega$; that is, when a subharmonic resonance exists. We consider the case of perfect time resonance and introduce a detuning parameter σ for the spatial part according to

$$k = 2K + \varepsilon\sigma, \quad \sigma = O(1), \tag{51}$$

$$\omega = 2\Omega. \tag{52}$$

Using Eqs. (51) and (52), we obtain

$$(kx_0 - \omega t) - (\bar{K}x_0 - \Omega t) = (Kx_0 - \Omega t) + (\sigma x_1 + 2iK_i x_0), \tag{53}$$

where K_i is the imaginary part and $\bar{K}$ is the complex conjugate of K.

To determine B, we seek a particular solution for the first-order problem in the form

$$u_1 = z_1(y;x_1)\exp[i(Kx_0 - \Omega t)] + cc, \tag{54}$$

$$v_1 = z_2(y;x_1)\exp[i(Kx_0 - \Omega t)] + cc, \tag{55}$$

$$p_1 = z_3(y;x\ )\exp[i(Kx_0 - \Omega t)] + cc. \tag{56}$$

Substituting Eqs. (35)-(37), Eqs. (43)-(45) and Eqs. (51)-(56) into Eqs. (30)-(34) and equating the coefficients of $\exp[i(Kx_0 - \Omega t)]$, we obtain

$$\mathcal{L}_1(z_1,z_2;K) = g_1, \tag{57}$$

$$\mathcal{L}_1(z_1,z_2,z_3;K,\Omega) = g_2, \tag{58}$$

$$\mathcal{L}_3(z_1,z_2,z_3;K,\Omega) = g_3, \tag{59}$$

$$z_1 = z_2 = 0 \quad \text{at } y = 0, \tag{60}$$

$$z_1, z_2, z_3 \to 0 \quad \text{as } y \to \infty, \tag{61}$$

where g_1, g_2 and g_3 are defined in Appendix A.

To obtain the solvability condition, we multiply Eqs. (57)-(59) by $\overset{*}{\zeta}_{21}$, $\overset{*}{\zeta}_{22}$ and $\overset{*}{\zeta}_{23}$, respectively, where the ζ^*'s are the solutions of the adjoint homogeneous problem, add the equations and integrate the resulting equation by parts from $y = 0$ to $y = \infty$. The adjoint problem corresponding to the eigenvalue K is

$$iK\overset{*}{\zeta}_{22} - D\overset{*}{\zeta}_{23} = 0, \tag{62}$$

$$i(U_0K - \Omega)\overset{*}{\zeta}_{23} + \overset{*}{\zeta}_{22}DU_0 - D\overset{*}{\zeta}_{21} - \frac{1}{R}(D^2 - K^2)\overset{*}{\zeta}_{23} = 0, \tag{63}$$

$$i(U_0K - \Omega)\overset{*}{\zeta}_{22} + iK\overset{*}{\zeta}_{21} - \frac{1}{R}(D^2 - K^2)\overset{*}{\zeta}_{22} = 0, \tag{64}$$

$$\overset{*}{\zeta}_{22} = \overset{*}{\zeta}_{23} = 0 \quad \text{at } y = 0, \tag{65}$$

$$\overset{*}{\zeta}_{21}, \overset{*}{\zeta}_{22}, \overset{*}{\zeta}_{23} \to 0 \quad \text{as } y \to \infty. \tag{66}$$

Then, the solvability condition can be expressed as

$$\int_0^\infty (g_1\overset{*}{\zeta}_{21} + g_2\overset{*}{\zeta}_{22} + g_3\overset{*}{\zeta}_{23})dy = 0. \tag{67}$$

Substituting for g_1, g_2 and g_3 from Appendix A into Eq. (67), we obtain the following differential equation for the evolution of B:

$$\frac{dB}{dx_1} = \frac{f_2}{f_1} A\bar{B}\exp(i\sigma x_1 - 2K_i x_0), \tag{68}$$

where f_1 and f_2 are given in quadratures in terms of ζ_{2n}, $\overset{*}{\zeta}_{2n}$, k and K and they are defined in Appendix B. To account for the parallel growth rate, we let

$$b = B\exp(-K_i x) \tag{69}$$

and substitute Eq. (69) into Eq. (68) to obtain

$$\frac{db}{dx_0} = -K_i b + \frac{f_2}{f_1}\varepsilon A\bar{b}\exp[(i\varepsilon\sigma - 2K_i)x_0]. \tag{70}$$

So far, the flow is assumed to be parallel. To account for the nonparallel effects only, we follow Nayfeh, Saric and Mook[6] and Saric and Nayfeh[7] and obtain

$$\frac{db}{dx_0} = (\frac{f_3}{f_1} - K_i)b, \tag{71}$$

where f_3 is defined in Appendix B. Hence, to account for the combined effects of the subharmonic resonance and nonparallelism, we have

$$\frac{db}{dx} = (\frac{f_3}{f_1} - K_i)b + \frac{f_2}{f_1}\varepsilon A\bar{b}\exp(i\varepsilon\sigma - 2K_i)x] \tag{72}$$

Note that when the nonparallel effects are neglected (i.e., $f_3 = 0$) Eq. (72) reduces to Eq. (70). Moreover, when the effects of subharmonic resonance are neglected, (i.e., $f_2 = 0$) Eq. (72) reduces to Eq. (71). Thus, Eq. (72) describes the nonparallel spatial growth of the amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode.

IV. Computational Procedure

A. Solutions of the basic-state and the zeroth-order problems

The procedure for the solutions of the basic state and the zeroth-order problem, given by Eqs. (38)-(42) and Eqs. (46)-(50), respectively, are the same. Therefore, we will here explain the methodology only for the basic-state problem.

Equations 38-42 can be expressed as a system of first-order differential equations; that is,

$$\frac{dz}{dy} = Gz, \tag{73}$$

where z is a 4x1 matrix with the elements

$$z_1 = \zeta_{11}(y), \; z_2 = D\zeta_{11}(y), \quad z_3 = \zeta_{12}(y), \; z_4 = \zeta_{13}(y), \tag{74}$$

and the elements of the 4x4 G matrix are given in Appendix C.

To determine starting solutions for the integration of Eqs. (73), we assume that $U_0 = 1$, $DU_0 = 0$ and $D^2U_0 = 0$ at $y = y_0$ with y_0 being any value of y larger than the boundary layer thickness. The matrix G then has constant coefficients at $y = y_0$ and Eqs. (73) have solutions of the form

$$z_i = \sum_{j=1}^{4} c_{ij}\exp(\lambda_j y) \quad \text{for } i = 1,2,3,4 \;, \tag{75}$$

where the c_{ij} are constants, the λ's are the solutions of

$$|G - \lambda I| = 0, \tag{76}$$

and I is the identity matrix. Equation (76) has the roots

$$\lambda_{1,2} = \pm\, k, \quad \lambda_{3,4} = \pm\, [k^2 + i(k - \omega)R]^{1/2} \;. \tag{77}$$

Two of these roots have positive real parts and make the solution grow exponentially as $y \to \infty$ and must be discarded according to the boundary conditions. This leaves two linear independent solutions that decay exponentially with y. To use SUPORT, we express the boundary conditions at $y = y_0$ in the form

$$Qz = 0 \quad \text{at } y = y_0, \tag{78}$$

where Q is a 2x4 matrix with constant coefficients and given in Appendix C.

The eigenvalues are not known a priori and must be determined along with the eigenfunctions. For given values of ω and R, we guess a value for k, generate the matrix G at $y = y_0$ and integrate the system of equations from $y = y_0$ to $y = 0$. If the guessed value of k does not satisfy the boundary conditions at $y = 0$, k is incremented by using a Newton-Raphson scheme and the procedure is repeated until the boundary conditions are satisfied. Integration is done by using a technique developed by Scott and Watts[8]. This technique orthonormalizes the solution of the set of equations whenever a loss of independence is detected.

B. Solution of the adjoint problem

The solution procedure is exactly the same as for the solution of the basic-state problem. The coefficients of the z matrix are

$$z_1 = \overset{*}{\zeta}_{22}(y),\ z_2 = D\overset{*}{\zeta}_{22}(y),\ z_3 = \overset{*}{\zeta}_{23}(y),\ z_4 = \overset{*}{\zeta}_{21}(y), \tag{79}$$

and the adjoint problem has the same eigenvalues as the zeroth-order problem.

C. Solution of the solvability condition

The calculations are repeated at different x locations to evaluate f_1, f_2, f_3, k and K for a given frequency along the x-axis. A fourth-order fixed step size Runge-Kutta integration scheme is used to solve Eq. (72) to find the amplitude of the subharmonic mode for different initial amplitudes of the T-S mode.

V. Numerical Results and Concluding Remarks

Computations were performed for three different dimensionless frequencies of the subharmonic wave, $F = 44 \times 10^{-6}$, 52×10^{-6} and 60×10^{-6}, by using different values of ε, where $F = \Omega/R = \Omega^* \nu/U_\infty^2$ with Ω being the dimensionless boundary-layer frequency and Ω^* being the dimensional frequency of the subharmonic wave. Here, ε along with the amplitude function A, which is not a constant for nonparallel flows, determines the amplitude of the fundamental wave. The amplitude function A can be calculated by using the usual nonparallel theory[6,7] and the calculated values of A for different frequencies are shown in Fig. 1. If A is normalized at the initial location, ε then represents the initial amplitude of the fundamental wave as a fraction of the mean flow. The calculations are started at the first neutral stability point of the fundamental wave and continued further downstream well into the unstable region of the subharmonic mode. The amplitude of the subharmonic mode, $b(x)$, is also normalized at the initial starting point.

Figures 2, 3 and 4 illustrate the variations of $b(x)$ as functions of R and ε for three different values of F. The solid curves are for the case of no interaction between the fundamental and the subharmonic waves. The point where the solid curve has a minimum is the first neutral stability point of the subharmonic mode.

The results show that the presence of the fundamental mode causes the amplitude of the subharmonic mode to oscillate before reaching its unstable region. As the subharmonic wave approaches its unstable region, it starts to amplify at a faster rate. Finally, it breaks away from the oscillatory growth region and amplifies considerably. The amplification of the subharmonic mode increases with increasing values of ε.

The amplification rate of the subharmonic wave can be much faster than that of the fundamental wave. However, as the amplitude of the subharmonic builds up, one can no longer neglect its effect on the fundamental wave and a nonlinear analysis is necessary to account for the interaction.

ACKNOWLEDGEMENTS

This work was supported by the United States Office of Naval Research Contract No. NR 061-201 and the NASA Langley Research Center Grant No. NSG 1255.

APPENDIX A

$$g_1 = -\frac{dB}{dx_1}\zeta_{21} \tag{A1}$$

$$g_2 = -U_0\frac{dB}{dx_1}\zeta_{21} - \frac{dB}{dx_1}\zeta_{23} + \frac{2iK}{R}\frac{dB}{dx_1}\zeta_{21} - [i(k - \bar{R})\zeta_{11}\bar{\zeta}_{21}$$

$$+ \zeta_{12}D\bar{\zeta}_{21} + D\zeta_{11}\bar{\zeta}_{22}]A\bar{B}\exp(i\sigma x_1 - 2K_i x_0) \tag{A2}$$

$$g_3 = -U_0\frac{dB}{dx_1}\zeta_{22} + \frac{2iK}{R}\frac{dB}{dx_1}\zeta_{22} - [-\bar{R}\zeta_{11}\bar{\zeta}_{22} + ik\zeta_{12}\bar{\zeta}_{21}$$

$$+ \zeta_{12}D\bar{\zeta}_{22} + D\zeta_{12}\bar{\zeta}_{22}]A\bar{B}\exp(i\sigma x_1 - 2K_i x_0) \tag{A3}$$

APPENDIX B

$$f_1 = \int_0^\infty [- \zeta_{21}\zeta_{21}^* - (U_0\zeta_{21} + \zeta_{23})\zeta_{22}^* - U_0\zeta_{22}\zeta_{23}^* + \frac{2iK}{R}(\zeta_{21}\zeta_{22}^*$$

$$+ \zeta_{22}\zeta_{23}^*)]dy \tag{B1}$$

$$f_2 = \int_0^\infty [i(k - \bar{K})\zeta_{11}\bar{\zeta}_{21} + \zeta_{12}D\bar{\zeta}_{21} + D\zeta_{11}\bar{\zeta}_{22}]\zeta_{22}^*$$

$$+ [- i\bar{K}\zeta_{11}\bar{\zeta}_{22} + ik\zeta_{12}\bar{\zeta}_{21} + \zeta_{12}D\bar{\zeta}_{22} + D\zeta_{12}\bar{\zeta}_{22}]\zeta_{23}^* \, dy \tag{B2}$$

$$f_3 = \varepsilon_2 \int_0^\infty [\frac{\partial \zeta_{21}}{\partial x_2}\zeta_{21}^* + \zeta_{22}^*(U_0 \frac{\partial \zeta_{21}}{\partial x_2} + \frac{\partial U_0}{\partial x_2}\zeta_{21} + V_0 D\zeta_{21} + \frac{\partial \zeta_{23}}{\partial x_2})$$

$$+ (U_0 \frac{\partial \zeta_{22}}{\partial x_2} + V_0 D\zeta_{22} + \zeta_{22} DV_0)\zeta_{23}^* - \frac{2iK}{R}(\frac{\partial \zeta_{21}}{\partial x_2}\zeta_{22}^* + \frac{\partial \zeta_{22}}{\partial x_2}\zeta_{23}^*)$$

$$- \frac{2i}{R}\frac{dK}{dx_2}(\zeta_{21}\zeta_{22}^* + \zeta_{22}\zeta_{23}^*)]dy \tag{B3}$$

where $x_2 = \varepsilon_2 x_0$ and $\varepsilon_2 = R^{-1}$ expressing the slight nonparallelism of the flow.

APPENDIX C

$$g_{11} = 0, \quad g_{12} = 1, \quad g_{13} = 0, \quad g_{14} = 0 \qquad \text{(C1)}$$

$$g_{21} = i(U_0k - \omega)R + k^2, \quad g_{22} = 0, \quad g_{23} = R\,\frac{dU_0}{\partial y}, \quad g_{24} = ikR \qquad \text{(C2)}$$

$$g_{31} = -\,ik, \quad g_{32} = g_{33} = g_{34} = 0 \qquad \text{(C3)}$$

$$g_{41} = 0, \quad g_{42} = -ik/R, \quad g_{43} = -\,[i(U_0k - \omega) + k^2/R], \quad g_{44} = 0$$

(C4)

Q is a 2x4 matrix consisting of the last two rows of the matrix B^{-1}.

The matrix B has the elements:

$$b_{11} = b_{12} = b_{13} = b_{14} = 1 \qquad \text{(C5)}$$

$$b_{21} = -\,k, \quad b_{22} = \tilde{k}, \quad b_{23} = k, \quad b_{24} = -\,\tilde{k} \qquad \text{(C6)}$$

$$b_{31} = i, \quad b_{32} = ik/\tilde{k}, \quad b_{33} = -\,i, \quad b_{34} = -\,ik/\tilde{k} \qquad \text{(C7)}$$

$$b_{41} = (\omega/k - 1), \quad b_{42} = 0, \quad b_{43} = (\omega/k - 1), \quad b_{44} = 0 \qquad \text{(C8)}$$

where

$$\tilde{k} = [k^2 + i(k - \omega)R]^{1/2} \qquad \text{(C9)}$$

REFERENCES

1. I. S. Kachanov, V. V. Kozlov and V. Levchenko, Mechanics of Liquids and Gases (in Russian) 3,4 (1977).
2. E. V. Veasov, A. S. Ginevsky and R. K. Karavosov, in Turbulent Flows (Nauka, Moscow, 1977, Ed. V. V. Struminskii, in Russian), p. 90.
3. P. S. Klebanoff, K. D. Tidstrom and L. M. Sargent, J. Fluid Mech. 12, 1 (1962).
4. C. von Kerczek, Presented at Twelfth Symposium on Naval Hydrodynamics, June 5-9, 1978, Washington D. C.
5. A. H. Nayfeh, Perturbation Methods (Wiley, New York, 1973), Chap. 6.
6. A. H. Nayfeh, W. S. Saric and D. T. Mook, Arch. Mech. Stosow 26, 401 (1974).
7. W. S. Saric and A. H. Nayfeh, Phys. Fluids 18, 945 (1975).
8. M. R. Scott and H. Watts, SIAM J. of Num. Analysis 14, 40 (1977).

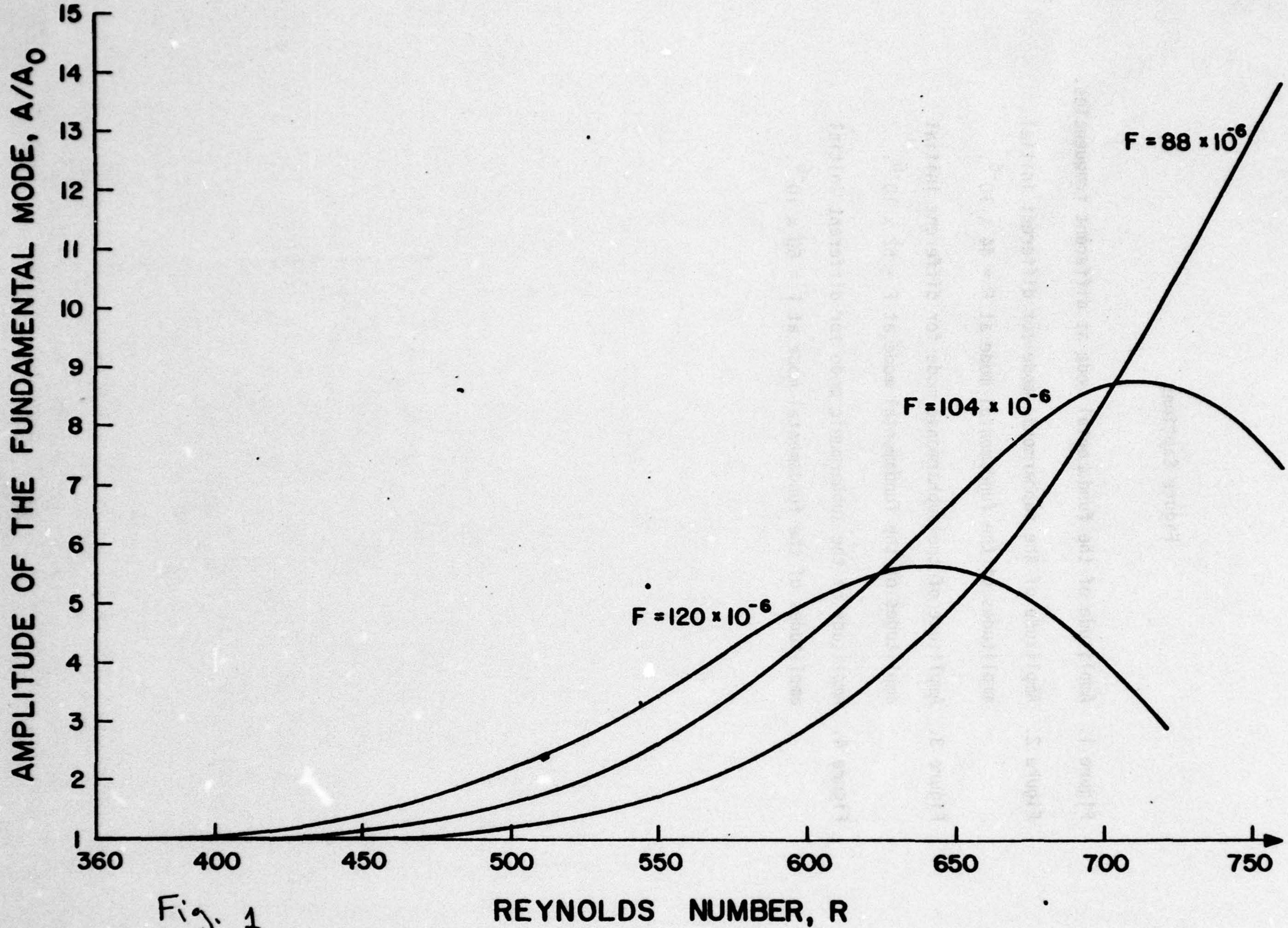
Figure Captions

Figure 1. Amplitude of the fundamental mode at different frequencies.

Figure 2. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 44 \times 10^{-6}$.

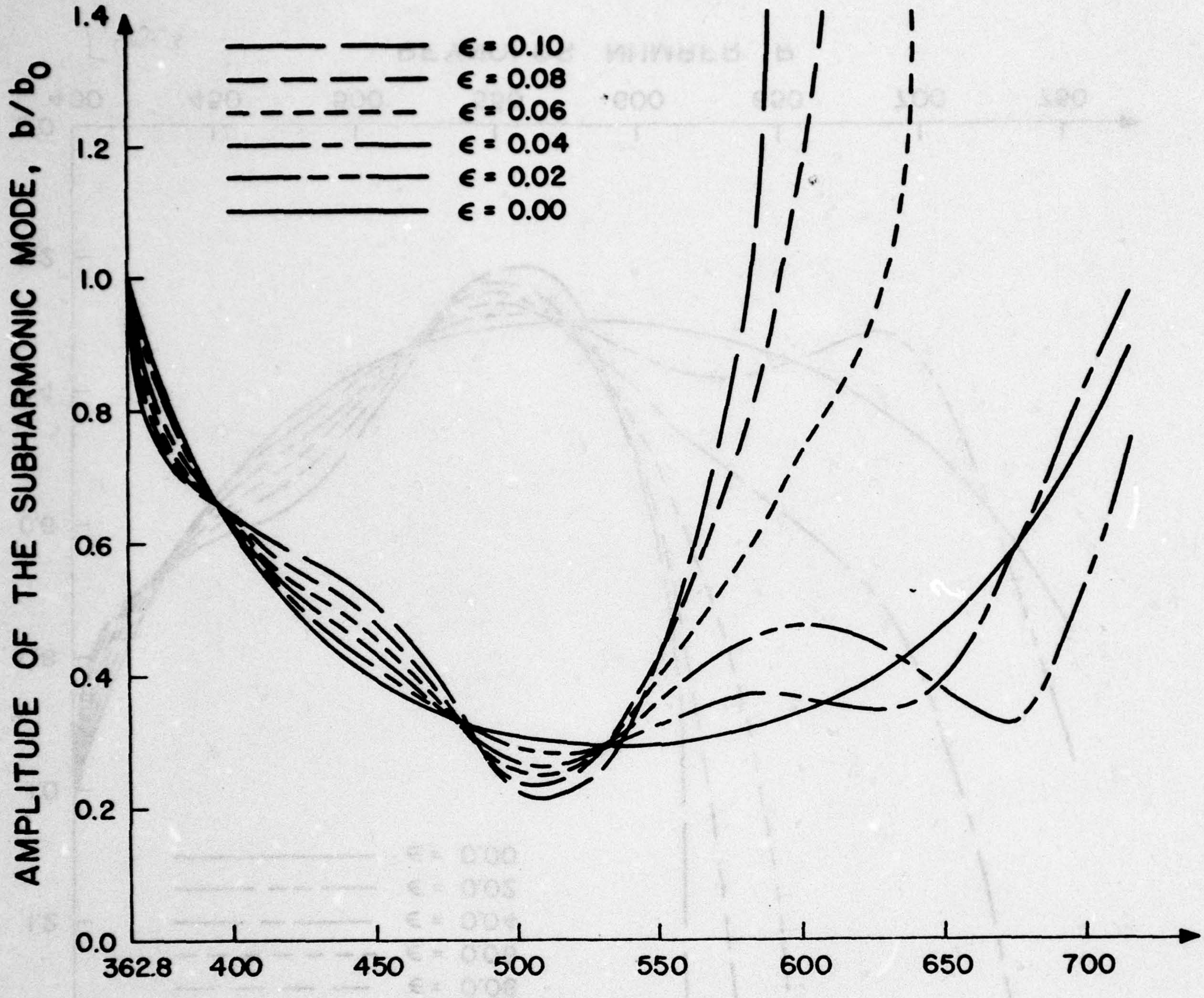
Figure 3. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 52 \times 10^{-6}$.

Figure 4. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 60 \times 10^{-6}$.



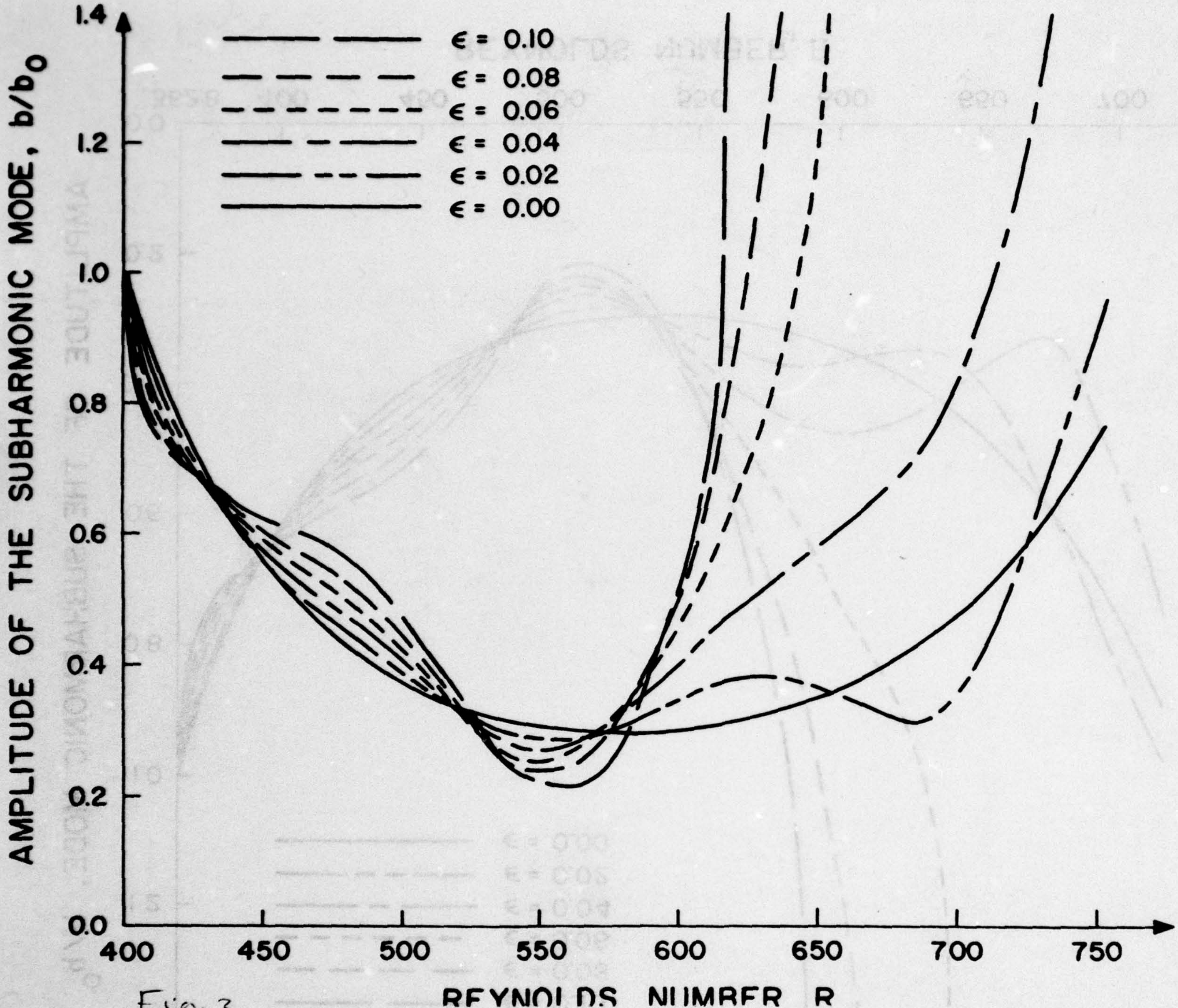
AMPLITUDE OF THE FUNDAMENTAL MODE, A/A0
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
360
400
450
500
550
600
650
700
750
REYNOLDS NUMBER, R
F = 88 x 10-6
F = 104 x 10-6
F = 120 x 10-6

Fig. 1



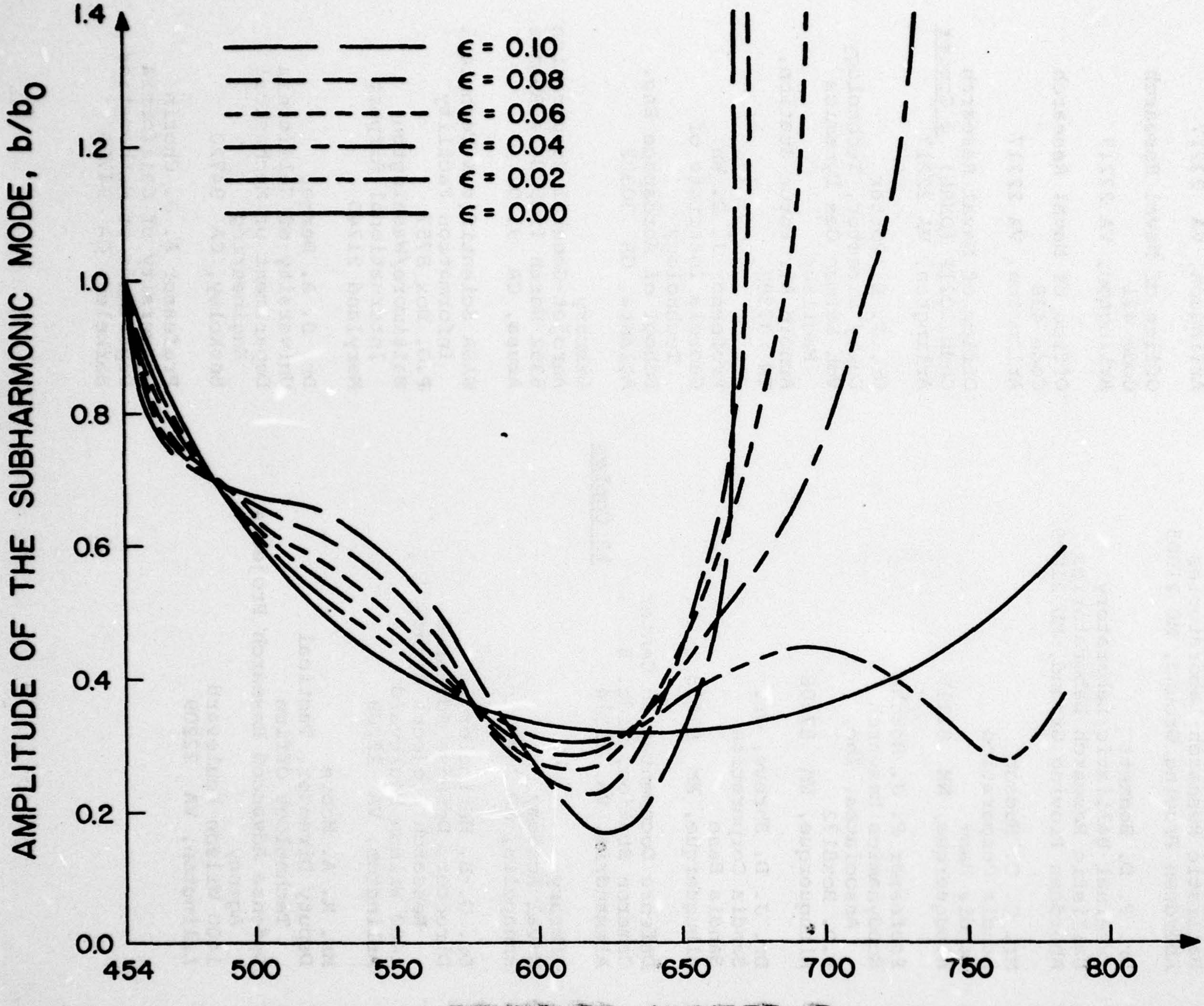
AMPLITUDE OF THE SUBHARMONIC MODE, b/b_0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\epsilon = 0.10$
$\epsilon = 0.08$
$\epsilon = 0.06$
$\epsilon = 0.04$
$\epsilon = 0.02$
$\epsilon = 0.00$
362.8
400
450
500
550
600
650
700
REYNOLDS NUMBER, R

Fig. 2



ϵ = 0.10
ϵ = 0.08
ϵ = 0.06
ϵ = 0.04
ϵ = 0.02
ϵ = 0.00
AMPLITUDE OF THE SUBHARMONIC MODE, b/b_0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
400
450
500
550
600
650
700
750
REYNOLDS NUMBER R

Fig. 3



AMPLITUDE OF THE SUBHARMONIC MODE, b/b_0
REYNOLDS NUMBER, R
$\epsilon = 0.10$
$\epsilon = 0.08$
$\epsilon = 0.06$
$\epsilon = 0.04$
$\epsilon = 0.02$
$\epsilon = 0.00$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
454
500
550
600
650
700
750
800

Fig. 4

Technical Library
Building 313
Ballistic Research Laboratories
Aberdeen Proving Ground, MD 21005

Dr. F. D. Bennett
External Ballistic Laboratory
Ballistic Research Laboratories
Aberdeen Proving Ground, MD 21005

Mr. C. C. Hudson
Sandia Corporation
Sandia Base
Albuquerque, NM 81115

Professor P. J. Roache
Ecodynamics Research
Associates, Inc.
P.O. Box8172
Albuquerque, NM 87108

Dr. J. D. Shreve, Jr.
Sandia Corporation
Sandia Base
Albuquerque, NM 81115

Defense Documentation Center
Cameron Station, Bldg. 5
Alexandria, VA 22314 12 Copies

Library
Naval Academy
Annapolis, MD 21402

Dr. G. H. Heilmeier
Director, Defense Advanced
Research Projects Agency
1400 Wilson Boulevard
Arlington, VA 22209

Mr. R. A. Moore
Deputy Director, Tactical
Technology Office
Defense Advanced Research Projects
Agency
1400 Wilson Boulevard
Arlington, VA 22209

Office of Naval Research
Code 411
Arlington, VA 22217

Office of Naval Research
Code 421
Arlington, VA 22217

Office of Naval Research
Code 438
Arlington, VA 22217

Office of Naval Research
Code 1021P (ONRL) 6 Copies
Arlington, VA 22217

Dr. J. L. Potter
Dupty Director, Technology
Von Karman Gas Dynamics
Facility
Arnold Air Force Station,
TN 37389

Professor J. C. Wu
Georgia Institute of
Technology
School of Aerospace Eng.
Atlanta, GA 30332

Library
Aerojet-General Corporation
6352 North Irwindale Avenue
Azusa, CA 91702

NASA Scientific & Technical
Information Facility
P.O. Box 8757
Baltimore/Washington
International Airport
Maryland 21240

Dr. S. A. Berger
University of California
Department of Mechanical
Engineering
Berkeley, CA 94720

Professor A. J. Chorin
University of California
Department of Mathematics
Berkeley, CA 94720

Professor M. Holt
University of California
Department of Mechanical Engineering
Berkeley, CA 94720

Dr. L. Talbot
University of California
Department of Mechanical Engineering
Berkeley, CA 94720

Dr. H. R. Chaplin
Code 16
David W. Taylor Naval Ship Research
and Development Center
Bethesda, MD 20084

Code 1800
David W. Taylor Naval Ship Research
and Development Center
Bethesda, MD 20084

Code 5643
David W. Taylor Naval Ship Research
and Development Center
Bethesda, MD 20084

Dr. G. R. Inger
Virginia Polytechnic Institute
and State University
Department of Aerospace Engineering
Blacksburg, VA 24061

Professor A. H. Nayfeh
Virginia Polytechnic Institute
and State University
Department of Engineering Science
and Mechanics
Blacksburg, VA 24061

Indiana University
School of Applied Mathematics
Bloomington, IN 47401

Director
Office of Naval Research Branch Office
495 Summer Street
Boston, MA 02210

Supervisor, Technical Library Section
Thiokol Chemical Corporation
Wasatch Division
Brigham City, UT 84302

Dr. G. Hall
State University of New York
at Buffalo
Faculty of Engineering and
Applied Science
Fluid & Thermal Sciences
Laboratory
Buffalo, NY 14214

Mr. R. J. Vidal
Calspan Corporation
Aerodynamics Research Dept.
P.O. Box 235
Buffalo, NY 14221

Professor R. F. Probstein
Massachusetts Institute of
Technology
Department of Mechanical
Engineering
Cambridge, MA 02139

Director
Office of Naval Research
Branch Office
536 South Clark Street
Chicago, IL 60605

Code 753
Naval Weapons Center
China Lake, CA 93555

Mr. J. Marshall
Code 4063
Naval Weapons Center
China Lake, CA 93555

Professor R. T. Davis
University of Cincinnati
Department of Aerospace
Engineering & Applied Mech.
Cincinnati, Oh 45221

Library MS 60-3
NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

Dr. J. D. Anderson, Jr.
Chairman, Department of
Aerospace Engineering
College of Engineering
University of Maryland
College Park, MS 20742

Professor W. L. Melnik
University of Maryland
Department of Aerospace Engineering
Glenn L. Martin Institute of Technology
College Park, MD 20742

Professor O. Burggraf
Ohio State University
Department of Aeronautical & Astronautical Engineering
1314 Kinnear Road
Columbus, Ohio 43212

Technical Library
Naval Surface Weapons Center
Dahlgren Laboratory
Dahlgren, VA 22448

Dr. F. Moore
Naval Surface Weapons Center
Dahlgren Laboratory
Dahlgren, VA 22448

Technical Library 2-51131
LTV Aerospace Corporation
P.O. Box 5907
Dallas, TX 75222

Library, United Aircraft Corporation Research Laboratories
Silver Lane
East Hartford, CT 06108

Technical Library
AVCO-Everett Research Laboratory
2385 Revere Beach Parkway
Everett, MA 02149

Professor G. Moretti
Polytechnic Institute of New York
Long Island Center
Department of Aerospace Engineering and Applied Mechanics
Route 110
Farmingdale, NY 11735

Professor S. G. Rubin
Polytechnic Institute of New York
Long Island Center
Department of Aerospace Engineering and Applied Mechanics
Route 110
Farmingdale, NY 11735

Technical Documents Center
Army Mobility Equipment R&D Center, Bldg. 315
Fort Belvoir, VA 22060

Dr. W. R. Briley
Scientific Research Assoc., Inc
P.O. Box 498
Glastonbury, CT 06033

Library (MS 185)
NASA Langley Research Center
Langley Station
Hampton, VA 23665

Dr. S. Nadir
Northrop Corporation
Aircraft Division
3901 West Broadway
Haethorne, CA 90250

Professor A. Chapmann
Chairman, Mechanical Engineerir Department
William M. Rice Institute
Box 1892
Houston, Texas 77001

Dr. F. Lane
KLD Associates, Inc.
7 High Street
Huntington, NY 11743

Technical Library
Naval Ordnance Station
Indian Head, MD 20640

Professor D. A. Caughey
Cornell University
Sibley School of Mechanical & Aerospace Engineering
Ithaca, NY 14853

Professor E. L. Resler
Cornell University
Sibley School of Mechanical & Aerospace Engineering
Ithaca, NY 14853

Professor S. F. Shen
Cornell University
Sibley School of Mechanical & Aerospace Engineering
Ithaca, NY 14853

Library
Midwest Research Institute
425 Volker Boulevard
Kansas City, MO 64110

Dr. M. M. Hafez
Flow Research, Inc.
P.O. Box 5040
Kent, WA 98031

Dr. E. M. Murman
Flow Research, Inc.
P.O. Box 5040
Kent, WA 98031

Dr. S. A. Orszag
Cambridge Hydrodynamics, Inc.
54 Baskin Road
Lexington, MA 02173

Professor T. Cebeci
California State University
Long Beach
Mechanical Engineering Department
Long Beach, CA 90840

Mr. J. L. Hess
Douglas Aircraft Company
3855 Lakewood Boulevard
Long Beach, CA 90808

Dr. H. K. Cheng
University of Southern California,
University Park
Department of Aerospace Engineering
Los Angeles, CA 90007

Professor J. D. Cole
University of California
Mechanics and Structures Department
School of Engineering and Applied
Science
Los Angeles, CA 90024

Engineering Library
University of Southern California,
Box 77929
Los Angeles, CA 90007

Dr. C. M. Ho
University of Southern California,
University Park
Department of Aerospace Engineering
Los Angeles, CA 90007

Dr. T. D. Taylor
The Aerospace Corporation
P.O. Box 92957
Los Angeles, CA 90009

Commanding Officer
Naval Ordnance Station
Louisville, KY 40214

Mr. B. H. Little, Jr.
Lockheed-Georgia Company
Department 72-74, Zone 369
Marietta, GA 30061

Dr. C. Cook
Stanford Research Institute
Menlo Park, CA 94025

Professor E. R. G. Eckert
University of Minnesota
241 Mechanical Eng. Bldg.
Minneapolis, MN 55455

Library
Naval Postgraduate School
Monterey, CA 93940

McGill University
Supersonic-Gas Dynamics Research
Laboratory
Department of Mechanical Eng.
Montreal 12, Quebec, Canada

Librarian
Engineering Library, 127-223
Radio Corporation of America
Morristown, NJ 07960

Dr. S. S. Stahara
Nielsen Eng. & Research, Inc.
510 Clyde Avenue
Mountain View, CA 94043

Engineering Societies Library
345 East 47th Street
New York, NY 10017

Professor A. Jameson
New York University
Courant Institute of Mathematical
Sciences
251 Mercer Street
New York, NY 10012

Professor G. Miller
New York University
Department of Applied Science
2636 Stuyvesant Street
New York, NY 10003

Office of Naval Research
New York Area Office
715 Broadway - 5th Floor
New York, NY 10003

Dr. A. Vaglio-Laurin
New York University
Department of Applied Science
2636 Stuyvesant Street
New York, NY 10003

Professor S. Weinbaum
Research Foundation of the City
University of New York on Behalf
of the City College
1411 Broadway
New York, NY 10018

Librarian, Aeronautical Library
National Research Council
Montreal Road
Ottawa 7, Canada

Lockheed Missiles & Space Company
Technical Information Center
3251 Hanover Street
Palo Alto, CA 94304

Director
Office of Naval Research Branch Office
1030 East Green Street
Pasadena, CA 91106

California Institute of Technology
Engineering Division
Pasadena, CA 91109

Library
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, CA 91103

Professor H. Liepmann
California Institute of Technology
Department of Aeronautics
Pasadena, CA 91109

Mr. L. I. Chasen, MGR-MSD Lib.
General Electric Company
Missile and Space Division
P.O. Box 8555
Philadelphia, PA 19101

Mr. P. Dodge
AiResearch Manufacturing Co.
of Arizona
Division of Garrett Corporation
402 S. 36th Street
Phoenix, Arizona 85034

Technical Library
Naval Missile Center
Point Mugu, CA 93042

Professor S. Bogdonoff
Princeton University
Gas Dynamics Laboratory
Department of Aerospace &
Mechanical Sciences
Princeton, NJ 08540

Professor S. I. Cheng
Princeton University
Department of Aerospace &
Mechanical Sciences
Princeton, NJ 08540

Dr. J. E. Yates
Aeronautical Research Assoc.
of Princeton, INc.
50 Washington Road
Princeton, NJ 08540

Professor J. H. Clarke
Brown University
Division of Engineering
Providence, RI 02912

Professor J. T. C. Liu
Brown University
Division of Engineering
Providence, RI 02912

Professor L. Sirovich
Brown University
Division of Applied Mathematics
Providence, RI 02912

Dr. P. K. Dai (RI/2178)
TRW Systems group, Inc.
One Space Park
Redondo Beach, CA 90278

Redstone Scientific Information Center
Chief, Document Section
Army Missile Command
Redstone Arsenal, AL 35809

U.S. Army Research Office
P.O. Box 12211
Research Triangle, NC 27709

Professor M. Lessen
The University of Rochester
Department of Mechanical Engineering
River Campus Station
Rochester, NY 14627

Editor, Applied Mechanics Review
Southwest Research Institute
8500 Culebra Road
San Antonio, TX 78228

Library and Information Services
General Dynamics-CONVAIR
P.O. Box 1128
San Diego, CA 92112

Dr. R. Magnus
General Dynamics-CONVAIR
Kearny Mesa Plant
P.O. Box 80847
San Diego, CA 92138

Mr. T. Brundage
Defense Advanced Research Projects Agency
Research and Development Field Unit
APO 146, Box 271
San Francisco, CA 96246

Office of Naval Research
San Francisco Area Office
760 Market Street - Room 447
San Francisco, CA 94102

Library
The Rand Corporation
1700 Main Street
Santa Monica, CA 90401

Department Librarian
University of Washington
Department of Aeronautics & Astronautics
Seattle, WA 98105

Dr. P. E. Rubbert
Boeing Commercial Airplane Co.
P.O. Box 3707
Seattle, WA 98124

Mr. R. Feldhuhn
Naval Surface Weapons Center
White Oak Laboratory
Silver Spring, MD 20910

Dr. G. Heiche
Naval Surface Weapons Center
Mathematical Analysis Branch
Silver Spring, MD 20910

Librarian
Naval Surface Weapons Center
White Oak Laboratory
Silver Spring, MD 20910

Dr. J. M. Solomon
Naval Surface Weapons Center
White Oak Laboratory
Silver Spring, MD 20910

Professor J. H. Ferziger
Stanford University
Department of Mechanical Engr.
Stanford, CA 94305

Professor K. Karamcheti
Stanford University
Department of Aeronautics and Astronautics
Stanford, CA 94305

Professor M. Van Dyke
Stanford University
Department of Aeronautics and Astronautics
Stanford, CA 94305

Engineering Library
McDonnell Douglas Corporation
Department 218, Bldg. 101
P.O. Box 516
St. Louis, MO 63166

Dr. R. J. Hakkinen
McDonnell Douglas Corporation
Department 222
P.O. Box 516
St. Louis, MO 63166

Dr. R. P. Heinisch
Honeywell, Inc.
Systems & Research Division - Aerospace Defense Group
2345 Walnut Street
St. Paul, MN 55113

Professor R. G. Stoner
Arizona State University
Department of Physics
Tempe, Arizona 85281

Dr. N. Malmuth
Rockwell International
Science Center
1049 Camino Dos Rios
P.O. Box 1085
Thousand Oaks, CA 91360

Rockwell International
Science Center
1049 Camino Dos Rios
P.O. Box 1085
Thousand Oaks, CA 91360

The Library
University of Toronto
Institute of Aerospace Studies
Toronto 5, Canada

Professor W. R. Sears
University of Arizona
Aerospace and Mechanical Engr.
Tucson, Arizona 85721

Professor A. R. Seebass
University of Arizona
Department of Aerospace and Mechanical Engineering
Tucson, Arizona 85721

Dr. S. M. Yen
University of Illinois
Coordinated Science Laboratory
Urbana, IL 61801

Dr. K. T. Yen
Code 3015
Naval Air Development Center
Warminster, PA 18974

Air Force Office of Scientific Research (SREM)
Building 1410, Bolling AFB
Washington, D. C. 20332

Chief of Research & Development
Office of Chief of Staff
Department of the Army
Washington, D. C. 20310

Library of Congress
Science & Technology Division
Washington, D. C. 20540

Director of Research (Code RR)
National Aeronautics and Space Administration
600 Independence Avenue, SW
Washington, D. C. 20546

Library
National Bureau of Standards
Washington, DC 20234

National Science Foundation
Engineering Division
1800 G Street, NW
Washington, D. C. 20550

Mr. W. Koven (AIR 03E)
Naval Air Systems Command
Washington, D. C. 20361

Mr. R. Siewert (AIR 320D)
Naval Air Systems Command
Washington, D. C. 20361

Technical Library Division (AIR 604
Naval Air Systems Command
Washington, D. C. 20361

Code 2627
Naval Research Laboratory
Washington, D. C. 20375

SEA 03512
Naval Sea Systems Command
Washington, D. C. 20362

SEA 09G3
Naval Sea Systems Command
Washington, D.C. 20362

Dr. A. L. Slafkosky
Scientific Advisor
Commandant of the Marine Corps
(Code AX)
Washington, D.C. 20380

Director
Weapons Systems Evaluation Group
Washington, D. C. 20305

Dr. P. Baronti
General Applied Science
Laboratories, Inc.
Merrick and Stewart Avenues
Westbury, NY 11590

Bell Laboratories
Whippany Road
Whippany, NJ 07981

Chief of Aerodynamics
AVCO Corporation
Missile Systems Division
201 Lowell Street
Wilmington, MA 01887

Research Library
AVCO Corporation
Missile Systems Division
201 Lowell Street
Wilmington, MA 01887

AFAPL (APRC)
AB
Wright-Patterson, AFB, OH 45433

Dr. Donald J. Harney
AFFDL/FX
Wright-Patterson, AFB, OH 45433

Commander, DCASMA, Phoenix
3800 North Central Avenue
Phoenix, Arizona 85012
Attention: GXCO

Mr. W. W. Trombley
The Garrett Corporation
Washington, D. C. 20006

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER VPI-E-78-19	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Secondary Instability in Boundary-Layer Flows		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ali H. Nayfeh and Ali N. Bozatli		8. CONTRACT OR GRANT NUMBER(s) NR 061-201 N00014-75-C-9381
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Polytechnic Institute and State University Engineering Science and Mechanics Department Blacksburg, Virginia 24061		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Program Director, Fluid Dynamics Office of Naval Research, Dept. of the Navy Arlington, Virginia 22217		12. REPORT DATE
		13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The stability of a secondary Tollmien-Schlichting wave, whose wavenumber and frequency are nearly one half of those of a fundamental Tollmien-Schlichting instability wave (T-S wave), is analyzed by using the method of multiples. Under these conditions, the fundamental wave acts as a parametric excitor for the secondary wave. When the amplitude of the fundamental wave is small, the amplitude of the secondary wave deviates slowly from its unexcited state. However, as the amplitude of the fundamental wave increases, so does the amplitude of the secondary wave even in the regions where it is damped in the unexcited state.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)